Endogenous Uncertainty: A Unified View of Market Volatility

by

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Abstract. The theory of Rational Belief Equilibria (RBE) offers a unified paradigm for explaining market volatility by the effect of "Endogenous Uncertainty" on financial markets. This uncertainty is propagated within the economy (hence "endogenous") by the beliefs of asset traders. The theory of RBE was developed in a sequence of papers assembled in a recently published book (Kurz [1997]) and the present paper provides a non-mathematical exposition of both the main ideas of the theory of RBE as well as a summary of the main results of the book regarding market volatility.

Section I starts by reviewing the standard assumptions underlying models of Rational Expectations Equilibria (REE) and their implications to market volatility. The paper then reviews four basic problems which have constituted puzzles or anomalies in relation to the assumptions of REE : (i) Why are asset prices much more volatile than their underlying fundamentals? (ii) The equity premium puzzle: why under REE the predicted riskless rate is so high and the equity risk premium so low? (iii) Why do asset prices exhibit the "GARCH" behavior without exogenous fundamental variables to explain it? (iv) the "Forward Discount Bias" in foreign exchange: why are interest rate differentials poor predictors of future changes in the exchange rates? Section II outlines the basic assumptions of the theory of RBE and the main propositions which it implies for market volatility. Section III develops the simulation models which are used to study the four problems above and explains that the domestic economy is calibrated, as in Mehra and Prescott [1985], to the U.S. economy. Then for each of the four problems the relevant simulation results of the RBE are presented and compared to the results predicted by a corresponding REE and to the actual empirical observations in the U.S.

The paper concludes that the main cause of market volatility is the dynamics of beliefs of agents. The theory of RBE shows that if agents disagree then the state of belief of each agent, represented by his conditional probability, *must fluctuate over time*. Hence the distribution of the individual states of belief in the market is the root cause of all phenomena of market volatility. The GARCH phenomenon of time varying variance of asset prices is explained in the simulation model by the presence of both *persistence* in the states of beliefs of agents as well as *correlation* among these states. Correlation makes beliefs either narrowly distributed (i.e. "consensus") or widely distributed (i.e. "non-consensus"). In a belief regime of consensus (due to persistence it remains in place for a while) agents seek to buy or sell the same portfolio leading to high volatility. In a belief regime of non-consensus there is a widespread disagreement which cause a balance between sellers and buyers leading to low market volatility. In short, the GARCH phenomenon is the result of shifts in the distribution of beliefs in the market induced by the dynamics of the individual states of belief.

Turning to the equity risk premium, the key question is what are the distributions of beliefs which ensure that the average riskless rate is low and the average equity risk premium is high. It turns out that *the only circumstances* when the mean riskless rate falls to around 1% and the mean equity premium rises to around 5.5% arise when, on the average, the majority of agents are relatively optimistic about the prospects of capital gains in the subsequent period. In such a circumstance the rationality of belief conditions imply that the pessimists (who are in the minority) must have *a higher intensity of pessimism than the intensity of the optimists*. In a large economy with this property the state of belief of any one agent may fluctuate but on the average there will be a *minority of intensely pessimistic agents*. This asymmetry between optimists and pessimists flows directly from the rationality conditions of beliefs and implies that at most dates the pessimists have a stronger impact on the bill market. At those dates the pessimists protect their wealth by increasing their purchases of the riskless bill. This bids up the price of the bill, lowers the riskless rate and results in a higher equity risk premium. In sum, the theory of Rational Belief offers a very simple explanation to the observed riskless rate and equity premium. It says that the riskless rate is, on average, low and the premium high because at most dates there is a minority of pessimist who*, by the rationality of belief conditions*, have the higher intensity level of belief about high stock prices in the future. These agents drive the riskless rate lower and the equity premium higher.

The "Forward Discount Bias" in foreign exchange markets is the result of the fact that in an RBE agents often make the wrong forecasts although they are right on the average. Hence, in an RBE the exchange rate fluctuates excessively due to the errors of the agents and hence at almost no date is the interest differential between two countries an unbiased estimate of the rate of depreciation of the exchange rate one period later. The bias is positive since agents who invest in foreign currency demand a risk premium on endogenous uncertainty which is above and beyond the risk which exists in an REE. The size of the bias is equal to the added risk premium due to endogenous uncertainty.

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I. The Basic Issue

This paper presents a unified view of market volatility that flows from the insight that volatility has two different components. One is generated by "fundamental" forces which are outside the economy and hence I refer to them as *exogenous.* The second is propagated within the economic system and I refer to it as the *endogenous* component. It follows from this perspective that understanding the nature of and causes for market volatility is useful for several reasons but here I want to stress three of them:

1. Understanding the distinction between the two components of volatility clarifies the nature of economic risk.
2. Fundamental information is useful only for the assessment and management of the exogenous and fundamental component of risk.
3. Understanding the endogenous component which is propagated within the economy is essential for assessing the nature and timing of investment opportunities.

Before explaining my theory, I briefly outline the perspective of the Market Efficiency Theory (as currently interpreted) or Rational Expectations[2] on these issues. My aim is not to compare in detail my theory with rational expectations but rather, to use rational expectations as an important *reference point for the evaluation of the problems which market volatility generates*. Also, my account is brief since rational expectations is the prevailing doctrine and most students of asset markets are familiar with it.

The standard formulation of an equilibrium of an economy and its financial markets starts with the dynamic portfolio and consumption choices of households and the production, investment and dividend decisions of firms. The theory is closed with market clearing conditions equalizing demands to supplies. Given the random nature of the

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[2] I shall use these terms interchangeably.

underlying economy it follows that equilibrium quantities (e.g. output, consumption, profits, prices, asset returns, etc.) are all stochastic processes with an underlying probability law. I call this probability the "true" probability law. Most of what is done in modern academic research in finance depends upon the utilization of this probability for computing objects like expectations (i.e. forecasts), theoretical covariances or for the Black-Scholes valuation of a derivative security. Thus, the idea that equilibrium is represented by a *true* stochastic process is fundamental to modern thinking in finance.

The rational expectations equilibrium (REE) theory is based on several assumptions, but three of them are fundamental to my discussion here. These are:

(A.1) The true probability law of the economy is stationary. In a *stationary* economy all the joint probabilities of economic variables remain the same as we move the time scale.

(A.2) Economic agents know the true probability law underlying the equilibrium variables of the economy. This is the first component of "structural knowledge" which the agents are *assumed* to possess.

(A.3) Agents know the demand and supply functions of all other agents. They can compute equilibrium prices of commodities and assets in the present and in the future given any possible exogenous fundamental information (i.e. news) in the future. This is the second component of structural knowledge which they possess.

I digress for a moment in order to discuss the relation of (A.3) to uncertainty. When formulating uncertainty in equilibrium, the standard theory specifies an *exogenous* "state space" which describes the totality of all that the agents are uncertain about with respect to the external environment. Examples of exogenous events that constitute an exogenous state include: weather conditions across the economy, earthquakes, machine breakdowns, fire destruction in the economy, health condition of the working population, births and deaths etc. In this paper I will use the term "news" to refer to such exogenous events but one needs to understand this term with great care. For example, a company's announced earnings is not the real exogenous state[3] but rather, it is a signal of the true *exogenous state of the company.* In the REE theory under (A.3) all the agents in the market know how to derive from the earnings report the true exogenous state of the company and this state is the *real* "news." I will return to the question of interpreting the

[3] The earnings report is clearly news in the sense of *newly observed information* in the public communication channels. It is not the *real* "news" which is defined to be the state of the exogenous environment that determines the earnings of the company.

exogenous states later since here I want to stress the implications of (A.3) for the nature of the implied equilibrium concept.

In equilibrium all economic magnitudes depend upon the realization of the exogenous state but according to (A.3) all agents know precisely the functional relations, or the map between equilibrium magnitudes (e.g. production decisions of firms, prices, dividend payments, returns, etc.) and the state. Consequently, all economic magnitudes vary only with the variability of the exogenous state over time. Moreover, it is then *an* assumption that given any observed information, all agents agree on the *meaning* or *interpretation* of such information. That is, all agents agree as to the "state" of the economy that gave rise to the observed information.

The implication of these assumptions is that *all* financial risks and observed volatility arise from causes which are *external to the economy* and I call such uncertainty "Exogenous Uncertainty". Under the above theory, no risk can be propagated from within the economic system via human beliefs or actions. This means that the volatility of equilibrium variables is exactly equal to the level that would be justified by the variability of the exogenous conditions.

The above discussion enables me to offer a simple summary of the conclusions of the theory of rational expectations with respect to the nature of market volatility:

1. For each state of the exogenous fundamentals there is a *correct* equilibrium price of all securities in the market.
2. If you possess all exogenous fundamental information you are able to compute the correct prices of securities and hence all uncertainty about prices will be resolved. By implication, hedging against the risks of all exogenous fundamentals is possible, in principle, and can control all risk associated with market volatility.
3. Active asset management has no function to play since the only investment management needed is the services of diversification and information gathering.

These conclusions of the theory have been at the foundation of contemporary research into the structure of market volatility. Unfortunately, they are in conflict with many theoretical and empirical observations and with common experience of market participants. Indeed, the implications of this theory have been rejected in broad areas of economics both on the empirical as well as on the theoretical levels. In order to discuss here specific issues I note that there are several outstanding problems or paradoxes (sometimes called "anomalies") related to the functioning of financial markets which the theory of rational expectations failed to resolve and current academic research has attempted to develop special theories to explain each one of these paradoxes. Since I will offer a *unified* view of market volatility, such a *single* theory would be more convincing if it could solve many of these problems. Here I focus on four central such problems and propose the ability of my theory to resolve these problems as a test of my perspective:

- **Problem A**: Why are asset prices and foreign exchange rates much more volatile than their underlying fundamentals?
- **Problem B**: Why do models based on rational expectations predict an equity risk premium over cash around .5% and a rate of return on cash of around 5% while over the last hundred years the average equity risk premium in the U.S. has been around 6% and the riskless rate has been in the range of .5% - 1%?
- **Problem C**: Why do asset prices exhibit the "GARCH" behavior of time varying variances when there are no fundamental factors to explain this phenomenon?
- **Problem D**: Why have interest rate differentials (between two countries) been such poor predictors of future changes in foreign exchange differentials in contrast with rational expectations, giving rise to the celebrated "Forward Discount Bias"?

I mention briefly an additional problem which follows from our earlier comments on exogenous uncertainty. The typical way of managing this uncertainty is by buying insurance: fire insurance, earthquake insurance, medical insurance, etc. If stockholders are risk averse and companies represent the interests of their stockholders, companies would insure against all available exogenous risks. Under rational expectations you can insure against future price variability by insuring against the exogenous risks that determine those prices. Hence, if a company buys insurance against all available risks why should the earnings of the company be random at all? Indeed, if all individual risks are reasonably independent across companies and households (or across companies and households that are spread over space), then insurance companies can provide insurance against most exogenous risks. Since insurance can drastically reduce the risk of earnings, equity ownership would not be risky in a rational expectations equilibrium and as a result of that the function of the stock market would be trivialized. This question will not be addressed in this paper. I raise it only in order to assist the reader in thinking through the question of whether all market risks *can* ultimately be traced to exogenous causes.[4]

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[4] Models of "Noisy" rational expectation equilibria have also attempted to address this problem within the rational expectations paradigm. In these models the noise in prices is assumed to be generated by the erratic trades of "noise traders" who are uninformed and irrational traders constituting a significant proportion of all traders in the market. I do not review this work in the present paper since it stands in sharp contrast to the basic rationality postulates of that paradigm. That is, since all the conclusions of a model of noisy rational expectations are driven by the arbitrary market actions of irrational traders, such a model should be viewed as a theory of irrational behavior with which one can prove anything. Also, from the empirical perspective it is hard to see who these noise traders are and since on average they lose money it is not clear what makes such traders survive.

Those who rejected the theory of rational expectations have tended to drift in diverse directions. Some have concluded that financial markets are dominated by investors who perceive probabilities incorrectly or are vulnerable to the impact of fads and mass psychology. Others have concluded that for some unexplained reason the market can be irrational *sometimes* and each failed prediction of the theory has been ascribed to a corresponding incident of such irrationality. As a result of such thinking, it is common to find in the investment community the argument that each instant of such *presumed* irrationality offers an opportunity for excess returns (i.e. when an investment opportunity is recommended as "excellent" and inexpensive). These perspectives are in conflict with the general view that there is logic and order in the market and therefore it should be possible to find one explanation for all these phenomena. This is my motivation for seeking a *unified* theory to provide a single tool for the study of market volatility.

I proceed by reviewing in Section II the basic premises of my new theory of Rational Beliefs and the allied concept of "Endogenous Uncertainty" which are the cornerstones of my approach. Section III, which is the main section of this paper, is devoted to showing via simulation results how the theory which I propose resolves the four Problems outlined. Most of the material presented here is based on papers published in a volume by Kurz (ed)[1997][5].

II. Endogenous Uncertainty and Rational Beliefs

II(A) *Rational Beliefs*

My theory of Rational Belief Equilibrium (RBE) developed in Kurz [1994a], [1994b] is based on the following alternative assumptions:

(AA.1) Despite the fact that the economy may undergo structural changes yielding non-stationarity, the economic universe is *stable* in the sense that statistical and quantitative analysis can be successfully carried out in it. In such a system the concept of "normal" patterns makes empirical sense and provides useful knowledge. It is represented by the long-term averages of economic variables. Thus, although our economy experiences technological and economic changes, the price/earning ratios of major indices have well known "normal" ranges and long-term (i.e. asymptotic) means, variances and covariances. Interest rates, growth

[5] Kurz (ed) [1997] *Endogenous Economic Fluctuations: Studies in the Theory of Rational Beliefs*. Studies in Economic Theory No. 6, Berlin and New York: Springer-Verlag, ISBN 3-540-62612-3. The introductory Chapter 1 (Kurz [1997a]) and the "Applications" Part B consisting of Chapters 9, 10, 11 and 12 contain the details which explain the ideas and support the results reported in the present paper.

rates, capital/output ratios etc. all have well known long-run average behavior which reveal some important dimensions of the true structure of the economy.

(AA.2) Economic agents do not know the true probability law underlying equilibrium magnitudes. This is the first component of structural knowledge which agents are assumed to lack.

(AA.3) Agents do not know the map from exogenous variables to equilibrium quantities in general and prices in particular. They have, however, access to the very large volume of all *past* data on the performance of the economy. This data they can use to statistically test any theory which they may develop about the functioning of the economy and of the financial markets. In this sense agents may learn something about structural relationships in the economy.

These assumptions ensure that although agents have no structural knowledge *they do have a common empirical knowledge*. I have already noted that a *stationary* economy is one in which all the joint probabilities of economic variables remain the same as we move the time scale. *Stationary systems are stable but stable systems are not necessarily stationary*. A system which experiences new technologies, new methods of production and new social organizations is not likely to be stationary but may be stable. This distinction is the central motive for the above assumptions and for this reason requires a detailed explanation.

Our economy is driven by a process of technological and organizational change which dominates every aspect of life in human history. This process is very complex but has a distinct character: once a new technology or organizational structure is established, it remains in place for some time until a new one is developed to replace it. While a technology or social organization is in place, the economy appears to have a fixed structure (i.e. it is stationary) until the next change. For simplicity I use the term "regime" to refer to such episodes in which the structure of the economy and the market are relatively fixed. Note that a regime in which steam ships dominate the technological frontier is very long and will have within it many, much shorter, sub-regimes. Moreover, the term may be used for the description of short periods in which a market may be dominated by a fixed configuration of factors, some fundamental and others involving the beliefs and perceptions of investors. In Figure 1a I give an example of such a sequence of regimes and the data which they generate. The horizontal bars represent the mean value functions which are constructed as constant within each regime. Figure 1b shows how we see the data without the knowledge of either the start and end dates of each regime or the mean value function prevailing within it.

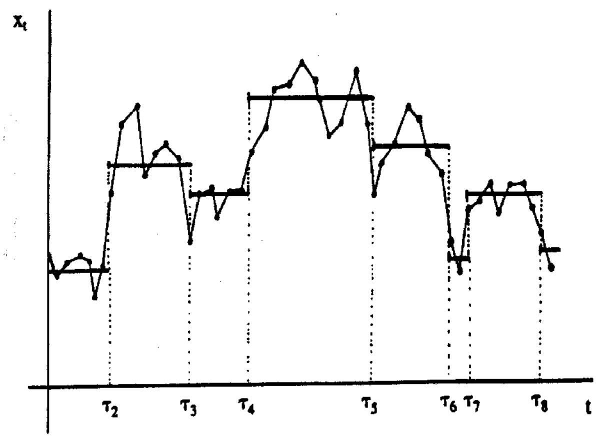

Figure 1a

Legend

(i) τ_j are dates of regime change
(ii) horizonal bars are mean value functions
(iii) data seen with parameters of structural change

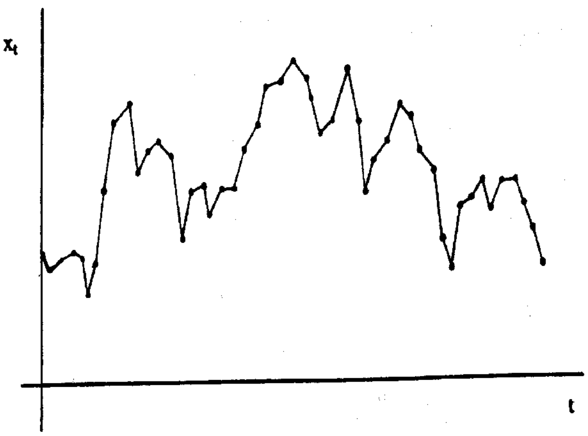

Figure 1b

data seen without any information about structural change

The important feature of a market characterized as a sequence of regimes is that in real time no one knows exactly the parameters of the prevailing regime or its starting and ending dates. Assumptions (AA.1)-(AA.3) aim to capture this reality. They do not deny the fact that if a regime lasts long enough investors will figure out approximately the character of the regime. Unfortunately, the fact that we can find out *in retrospect* the nature of the last regime does not mean that we learn the probability law of the entire future evolution of the process or that we can correctly predict the next regime. This explains why the first result derived from my assumptions (AA.1) and (AA.3) is:

> R(1) The true probability underlying the system cannot be learned and even if an agent discovers it, he cannot be sure that it is the true probability (or dynamic law of motion). Equally so, economic agents cannot learn the equilibrium map between market prices and those variables which determine prices. Such a map may change across regimes.

The development of the theory of Rational Beliefs is then based on the observation that agents who make investment decisions do not know the *true* probability of the processes of earnings and stock prices. Since they need to make investment decisions they must form their own separate theories or models of the process. Hence, out of the fact that the true probability of the market dynamics cannot be known for sure, emerges the conclusion that investors will disagree both in their forecasts of the future as well as in the interpretations of market news. The reader may find this conclusion self-evident, but it has important implications which will be explored in Section II(B) below.

Assumptions (AA.1)-(AA.3) also specify *what the agents do know* and this fact is the basis for the next development. Specifically, assumption (AA.3) means that all agents know the empirical distributions of past data and this common empirical knowledge provides the basis for a new definition of the *rationality* of beliefs.

A belief is then a theory or a subjective model of the market which takes the form of a system of joint probabilities over all relevant economic variables. Such a belief is called a *rational belief* if it cannot be contradicted by the statistical knowledge represented by the known empirical distributions. A rational belief must have the property that if one simulates the model with many runs over time it will generate statistics (i.e. and empirical distribution) which are exactly the same as those that were generated by the historical record of the market. Thus the concept of a rational belief isolates that subset of all possible theories or models that are *compatible* with the available data (i.e. that cannot be contradicted by it).

In my approach, the rationality of beliefs rests on the premise that the economic universe is stable so that two rational agents holding two different theories cannot disagree about the long run statistics (means, variances, covariances etc.) which both of their individual theories are required to "reproduce". If any model generates long term statistics which differ from the empirical evidence, it is judged wrong and the underlying belief are judged irrational. I will now explain other important results of the theory.

II(B) *Diversity of Beliefs and Mistakes in a Rational Belief Equilibrium*

A dynamically changing but stable economy is one in which economic variables may be transformed (e.g. into logs or into growth rates rather than absolute values if needed) so that although structural changes take place, all long term frequencies and averages converge. These frequencies and averages are learned by all agents and represent the "normal" probabilities of events. Investors often consult such information when they describe how frequently a certain pattern of events happened over the last two hundred years! An agent who believes that the world is stationary would adopt these normal frequencies as his belief and select his portfolio, investment and consumption decisions accordingly. This result can be summed up by:

R(2) The theory holds that an agent who adopts the normal empirical frequencies as his belief is entirely rational since his belief is compatible with the empirical distributions.

Note, however, that such a person must also believe that the joint probability distributions of economic and financial variables in the 1990's are the same as the joint distributions in the 1980's and both are equal, according to him, to the joint distributions computed as averages over many past years. That is, he believes that no structural changes ever take place or that technological or structural changes in the real economy have a *neutral* effect on financial markets and thus have no effect on the structure of market performance.

If the economic system is stationary and if all the agents *knew for sure* that it is stationary, then they will all learn the true probability law of motion and will know that this true law of motion is the one calculated from the empirical distributions of past events. They will also all agree on the correct pricing model of all assets.

In contrast, I have already expressed my view that the process of structural change (i.e. non-stationarity) in our society is *the* central building block of its complexity and the root cause of the diversity of beliefs about it. In such a system the past is not an entirely satisfactory basis for assessment of risks in the future and at every date many agents hold the view that the market and economy may be *similar* to the past but yet very different indeed. The implication is that an agent who forms a forecast which is different from the historical statistical average is adopting a sharper view of the future than can be deduced from the statistics of the past. Such a theory may not be contradicted by past data but past data is not required to support it either since the belief of the agent may be based on a model according to which the future is *different* from the past. That is, an agent who holds a theory of the market which insists that the situation today is *different* from the past cannot support his theory by the long run statistics of the past. He may offer some statistical evidence of *recent* developments to bolster his model but such evidence would lack high statistical reliability and thus may not be acceptable to other agents. His theory may sometimes be right and sometimes be wrong.

What is the patterns of disagreement among these rational agents? Motivated by the observations above, the theory of rational beliefs shows that:

> R(3) The main source of disagreement among agents derives from the fact that they can hold different theories both about the *nature* and *intensity* of changes in the economy as well as their *timing.* As a result, given commonly observed news at any date, agents can have very different opinions regarding the significance of the news to future market performance. For example, some may be optimistic while others are pessimistic about it.

The mere fact that agents disagree has an immediate and very important implication.

> R(4) A group of economic agents who hold rational beliefs and pairwise disagree

> forever (at all times and in the limit rather than have a one-time disagreement) must also experience variations in the probabilities with which they forecast future economic events at different dates. This means that the "states of belief" of these agents *must fluctuate over time.*

I stress that conclusion R(4) is a *consequence* of the theory of rational beliefs together with the observations that agents disagree. To understand why this conclusion holds note that if a group of agents disagree pairwise forever then all but one of them must not believe that the economy is stationary and hence they do not permanently adopt the normal frequencies as their beliefs. However, their beliefs must be compatible with the normal frequencies in the exact sense that *deviations of their one period probability beliefs from the normal frequencies must average to zero*. That is, if you are optimistic relative to the normal frequencies in some dates you must be pessimistic relative to those frequencies in other dates so that on average you expect your deviations from the normal frequencies to average to zero. But then it follows that all permanent disagreements imply variability in probability beliefs around the normal frequencies.

Let me examine the implication of R(4). It says that if we observe a market in which there is always some disagreement among agents who hold rational beliefs then their disagreements will not be fixed. If we study those disagreements we shall find that they will be the result of on-going reassessment. As a result, the states of beliefs of the disagreeing individuals will be changing over time. Note that this does not mean that the distribution of beliefs in the market as a whole will be changing over time as well. I return to this important subject when I discuss in V(iii) the results regarding the equity risk premium.

The dual requirement of stability and of compatibility with the empirical distributions impose restrictions on the models of the economy which a rational agent can adopt as his belief. Nevertheless, the theory allows sufficient heterogeneity of beliefs to persist over time so that the subjective models used by the agents may imply forecast functions which can be different for different agents at all dates. In short, my theory permits two intelligent investors who observe the same (and vast) information about the past to have different opinions and hence to make different forecasts of the future.

This brings me to an important observation. If there is a *true and unknown* equilibrium probabilistic law of motion underlying the dynamics of the market, and if there are substantial differences in probability beliefs among the agents about the future, then, although all the agents are rational, most may be holding wrong beliefs. This leads them to make *forecasting mistakes*. To clarify this point recall Figures 1a-1b which reveal the problem of an agent who forms a belief about the market. Suppose that the price/earnings ratio of an index of his interest is the highest in 40 years. If he follows the statistics of the long past he will compute the fact that, say, only in 7.8% of past cases the price/earnings ratio went higher than the observed level and hence the probability of capital gains is exactly 7.8%. With such probability the investor decides that the index is too high and his portfolio decision is to sell. Another investor, observing the identical information about prices and earnings, formulates a model about the future productivity of the firms in the index on the basis of which he concludes that the statistical record of the past is not completely applicable. Based on his model, he believes that the probability of higher prices is 60% on the basis of which his portfolio decision is to buy.

I suggest that one or both of the two investors hold wrong beliefs and are thus making a mistake. More formally, the *mistake* of an agent at date t is defined as the function which describes the difference between the collection of his forecasts at date t conditional upon the information at that date and the forecasts that would be made with the correct model, were it known. Since an agent selects his decisions (i.e. portfolio, investments, etc.) based on his beliefs, these mistakes in beliefs get translated into mistaken *actions.* In equilibrium, quantities and prices will reflect those mistakes. Thus, the economic variables which we observe at each date contain the mistakes of the agents and this fact will be the foundation of the concept of "Endogenous Uncertainty."

I caution against a simplistic interpretation of the term "mistake". In its daily use this term usually refers to acts or thoughts which are wrong but *which could have been avoided.* Here a "mistake" is a rule by which a rational agent utilizes information efficiently but fails to make the correct forecast. In fact, it is essential that there is no statistical way through which an agent can be assured of avoiding making a "mistake" in my sense. Thus, in the context of this theory *rational agents make mistakes.* The theory does not say that agents who form an opinion which deviates from the statistical norm be "certain" or sure of the truth of their model. What the agents do know is that without committing to an investment program that will take advantage of the changing conditions of the market, they cannot make excess returns.

My approach implies, therefore, that the nature of "risk assessment" by the agents is quite different from the usual analysis of the covariance structure among asset returns. For these agents the market is an arena for the competition among theories or "models" that seek to capture future excess returns. In such a market the risky nature of a decision is tied to a commitment to a theory of the market without having *statistically* reliable evidence in support of such a theory. "Assessment" of such risks has something to do with the way we *interpret* existing information rather than with a direct utilization of past covariances. This is particularly true in an environment of changing regimes where advanced (observed) signals about the coming regime are usually available, but agents have insufficient statistical evidence to be able to interpret such information with a high level of statistical reliability.

An economic equilibrium in which all agents hold rational beliefs is called a *Rational Belief Equilibrium* (RBE). In such an equilibrium the investment, consumption and portfolio decisions are, in part, determined by the mistakes of the agents and these effects can be substantial. The implication is that the mistakes of agents have an effect on equilibrium prices and on the real allocations in the economy. Alternatively, in an RBE the beliefs of agents have real effects on the performance of the economy; they influence the volatility of economic variables such as output, investment and prices. This leads to the fifth result:

> R(5) If individual agents can make mistakes in the assessment of market values, then the market as a whole can also evaluate assets "incorrectly". This conclusion should be understood in the sense that such pricing can be different from that pricing that would be justified by the true market forecast. Equilibrium market prices may overshoot above "fundamental values" when asset prices rise and may overshoot downward, when asset prices declines, below the values that would be justified by fundamental exogenous variables .

This conclusion shows that an important component of the volatility of economic variables is generated by the mistakes of agents and these arise from the variability in the states of beliefs of the agents. To see a simple example of why this could be important, suppose that some investors develop a theory according to which a particular imminent development may adversely affect the profits of some firm. The actions of these investors will induce a fall in the price of the shares of the firm with no exogenous event to "justify" it. Moreover, if the theory of these agents is wrong, prices will ultimately return to their original position and the entire move would have been induced only by the forecasting mistakes of the agents. Similar arguments would apply to other economic variables such as an investment by a firm or a purchase of foreign currency by a trader: beliefs and forecasting mistakes have real effects on the fluctuations of economic variables. That component of volatility of these variables above and beyond the level that would be justified by the exogenous variables of the system is therefore said to be *internally propagated*. I call this type of uncertainty *Endogenous Uncertainty*[6].

II(C) *Components of Endogenous Uncertainty*

Anticipating the developments in Section III below I briefly evaluate the specific

[6] This component of market uncertainty is called *Endogenous Uncertainty* in Kurz [1974]. In Kurz [1994b] I work out a simple Rational Belief Equilibrium that provides an example of an exact mechanism which propagates endogenous uncertainty in this manner.

factors which contribute to this component of market volatility. I think that most experienced market participants know too well that beliefs and expectations have an effect on market performance and will thus find the concept of endogenous uncertainty to be very natural. Hence, the evaluation below can be made with a practical perspective. Think of a market in which, at any date or over a period (which may constitute a regime), an agent holds a probability belief about future economic events which deviates from the normal pattern. For example, the agent may sometimes be relatively optimistic and sometimes relatively pessimistic about future increases of price/earnings ratios relative to the probability calculated on the basis of the historical experience. How would these levels of *relative* optimism and pessimism contribute to market volatility over time?

1. *The dynamics of beliefs*. Following the conclusion in R(4) this factor measures both the *frequency* at which the models of the agents call for change in their periodic outlook as well as the intensity of their deviation from the normal frequencies when their models call for such deviations. If the models of the agents induce rapid changes and each change calls for intensive response, the market impact of such models would be different from the impact of models which change slowly and call for a low intensity level of response.
2. *The distribution of beliefs*. Compare a distribution in which 5% of the agents are optimistic, 5% are pessimistic and 90% are neutral with a distribution in which 50% are optimists and 50% are pessimists. Although both distributions are "balanced," it is a fact that the latter *could cause a much higher level of market volatility than the former.*
3. *Correlation among beliefs*. It is evident that if the forecasts of a large number of market participants shift together in one direction or another, market volatility may be drastically affected. These are exactly the conditions which generate very high prices or low "crash prices." This component of the theory is defined by the correlation among the beliefs of the agents and is an important condition which shapes the nature of endogenous uncertainty. The correlation between two agents takes several forms:

(i) Agreement between the two agents on the *direction* of the deviation of their probability belief from the normal pattern at a given date.
(ii) Similarity of the *intensity* of the deviation of their probability belief from normal patterns when they agree on the direction of deviation.
(iii) Similarity in the *interpretation of market information* as a trigger for deviation from the normal frequencies. This factor amounts to agreement or disagreement on the interpretation of "news." The most important example of this factor is the

similarity in which the models of two agents select, at date t, their conditional probability beliefs about future prices in response to the realization of a market price at that date.

The simulation results of the next section address the four Problems formulated in Section I and the evaluation of the results will be carried out with the above components of correlation in mind. Before turning to the simulations I note that in an Appendix to this paper I report the results of two other studies (with no relation to the simulation work) which examine empirically the U.S. stock market. These studies test the theory of RBE and evaluate the *quantitative* role of endogenous uncertainty in the market. They show that the basic predictions of the theory are consistent with the data and that endogenous uncertainty accounts for more than half of all pure market uncertainty.

III Explaining the Paradoxes: Simulation Results

I have suggested to the reader that my theory offers a unified paradigm to solve the four problems formulated in Section I. Here I review these solutions in the form of simulation results of models with endogenous uncertainty. Since the questions span issues related not only to the domestic but also to the international economy, I present the results of two slightly different models: one of the domestic economy and a second of the international economy[7]. The two models have the same basic structure which I shall review first. The assumptions specified below highlight the main features which are studied while maintaining simplicity so that the models could be numerically solved. After this review I present the results and interpret them.

III(A) *The Basic Models*

The two models used in the simulations reported below consist of the following components:

- Both the domestic as well as the international models have a *single* stock market in which the ownership shares of business firms are traded. The earnings processes of the firms are exogenous stochastic processes. In the international model the stock market is in the "home" economy and the foreigners trade in the stock market of the home economy.

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[7] All numerical results for the domestic economy are developed in Kurz and Schneider [1996] and in Kurz and Beltratti [1997] who utilize the same model. The results for the international economy are in Kurz [1997c] and Black [1997]. The reader may consult these papers for details.

• The debt instrument in the domestic model is a *real* short term debt instrument (which I shall call a "bill") with a riskless return: at each date it is purchased at a discount and it pays in the subsequent period one unit of consumption. Hence, the domestic model has two financial markets: a stock market and a market for a riskless indexed real bill.

• The structure of debt instruments in the international model is much more complicated. Each country has a short term *nominal* "bill" which is purchased at each date at a discount and which pays in the subsequent date *one unit of currency* of the respective economy. These financial assets are not riskless since the owner of such instruments faces currency and price level risks due to the fact that these are nominal bills paying in units of currency in the respective countries. In addition, there is a *real* short term international riskless "bill" which is purchased at a discount at each date and which pays a unit of consumption in the next period. Hence, the international model has four financial assets: stocks, a nominal bill in the home economy, a nominal bill in the foreign economy and an internationally traded real (indexed) bill.

• The economy consists of multiple agents with an initial endowment of "wage income" which they need to trade at any date to purchase consumption and financial assets. In the home economy the only stochastic shock is the earnings process so that wage income is assumed to be riskless in the home economy. In the foreign economy I assume that wage income is a random variable. This last assumption is made for technical reasons and simplicity, but the force of the assumption is that in the international model each economy has only one exogenous shock.

• In the next period the agents of both economies receive the various financial payments specified above in addition to the fact that they collect capital gains on their stock holdings since the stocks are long lived securities.

• The growth rates of earnings of the firms and the growth rate of wage income in the foreign economy are both stochastic processes with the property of persistence (i.e. they are Markov processes). Hence, earnings in the home economy and wages in the foreign economy are informative variables for predicting future earnings and wages.

• Agents hold rational beliefs which recognize the persistence property of earnings and wages. However, they have individual models which result in the fact that at each date any one of the agents (or all) may be relatively optimistic or relatively pessimistic about high stock prices the next period. These beliefs may be correlated and the intensity of optimism or pessimism may vary with realized prices at each date. These assumptions of the simulation models and the effects of this assumed structure of beliefs will be described in further detail via a simple example developed for the explanation of the results in V(iii) below about the equity risk premium.

• The international model includes money and allows for monetary policies of the two economies. Since it is not my aim to study different monetary policies, I fix the policies in the two countries. They are set so that each country responds to its own exogenous shock: the domestic central bank adjusts the money supply in response to the random changes in the growth rate of earnings and the foreign central bank adjusts the money supply to changes in the growth rate of wages. In either country the objective of the bank is to maintain price stability.

III(B) *On the Method of Simulations*.

What is the logic of a simulation model and why should we consider this method of analysis valid? To answer this question I note first that the parameters of the real economy are selected so as to conform to well known parameters of econometric models that were estimated for the U.S. economy. These include the long term growth rates of wages and earnings and the coefficients of risk aversion and discount rates of the agents. As a result, the real part of the economy is required to act in conformity with what we know about the long run tendencies of the U.S. economy. Hence, *the fundamentals of the economy are exactly the same as we know from the statistics of the real economy.* The parameters which I, as a model builder, will select are those that relate to the beliefs of the agents and their distributions. The simulation models then ask what would be the implications of alternative belief structures for price volatility, *holding the fundamentals fixed*. Since rational expectations are among the beliefs which can be examined in the model, the results below will provide a comparison between the implications of rational expectations and rational beliefs to price volatility, keeping the real economy the same.

It has been well documented that if one imposes on the real fundamentals of the simulation models the assumption of rational expectations by the agents, all the problems and paradoxes specified earlier will appear and I shall demonstrate that this remains true in the models at hand. However, if I can show that under the assumption that the agents hold rational beliefs the financial markets will not exhibit any of the paradoxes, then it follows that the belief structure of the agents does provide a unified paradigm to resolve the specified problems. It would then be useful to have an intuitive understanding of the structure of beliefs that generate the various conclusions and I will attempt to provide some interpretation in a later section.

III(C) *Simulation Demonstration of the Solutions to the Four Problems*

In the Tables below I present comparisons between the simulation results under rational expectations and under rational beliefs. The aim is not to compare the two theories since a comparison of the two would require far more details than I have provided. Instead, my aim is first to exhibit what are the problems which arise under the currently prevailing paradigm in finance and then to show that these problems are significantly resolved under the unified paradigm of the theory of rational beliefs. The sequence of the tables below correspond to the questions posed at the start. In Section IV I provide an interpretation that will help the reader understand how the theory is applied.

C.1 <u>Problem A: Asset Price Volatility in the Domestic Economy</u>

Table 1 reports two measures of price volatility. The first is the interval in which the price/earnings ratio fluctuates 95% of the time. The long term mean of this variable is fixed at 13.9 which is the actual long term average of the price/earnings ratio of the S&P 500 index. This average has no significance in the table and is used only as a reference for measuring the interval of fluctuations under each of the model assumptions. The

<u>Table 1: Long Run Volatility of the Price/Earnings Ratio and the Return on Equity</u>

	Under Rational Expectations	Under Rational Beliefs	Actual
Interval in which the price/earnings ratio fluctuates 95% of the time	[13.8 , 14.0]	[9.4 , 18.4]	[5.5 , 22.3]
σ_r - the long term standard deviation of the return on equity	4.1%	17.5%	18.4%

second row in Table 1 reports the long term standard deviation of the real rate of return (corrected for inflation) on equity.

The table exhibits the problem which arises under rational expectations: if stock prices vary strictly in accord with fundamentals they would not change very much! The variance of the price/earning ratio is bigger *by an order of magnitude* under rational beliefs than under rational expectations. The table shows that under rational beliefs the index would have spent 95% of the time between 9.4 and 18.4 which is of the same order of magnitude as the historical record. This interval is somewhat smaller than the actual interval reported in the last column, a fact that may be explained by the generally agreed upon observation that the fluctuations of the reported price/earnings ratio are sensitive to tax and accounting practices. These tend to overstate the volatility of recorded earnings relative to the true economic earnings of the companies in the index. The actual long term standard deviation of the return on the S&P 500 index is 18.4% and the simulations under rational beliefs lead to a standard deviation of 17.5%. These two measures of volatility are very close.

C.2 Problem B: The Equity Premium and the Riskless Rate in the Domestic Economy

In Table 2 I record the long term averages of the riskless real rate of return on cash (corrected for inflation) and of the equity risk premium over cash. The table exhibits the problem which arises under rational expectations: the historical record over the last hundred years shows a real rate of return on cash in the order of magnitude of 1/2% - 1% and an average risk premium over cash of around 6%. The model under rational expectations fails to come close to these historical facts as seen in the table. Under rational beliefs the average equity premium is 6.46%, the average rate of return on cash is .53% and these figures correspond to the historical record.

Table 2: The Long Run Average Riskless Rate On Cash
and the Equity Premium Over Cash

	Under Rational Expectations	Under Rational Beliefs	Actual (Approx)
r^F - the long term average of the riskless rate on cash	5.04%	.53%	.50%
ρ - the long term average risk premium of equity over cash	.57%	6.46%	6.00%

C.3 Problem C: The GARCH Property of Stock Prices in the Domestic Economy

It has been observed both by experienced market traders as well as by academic researchers that over time, the variance of stock prices changes *without* a corresponding change in fundamentals to account for it. This is known as "the GARCH property of stock prices" and this represents a problem for rational expectations since under such expectations prices change only in response to changes in fundamentals. In Figure 2 I exhibit a plot of the time series of 300 prices that were simulated in the domestic model. The growth rate of earnings is assumed to take two values in these calculations and since

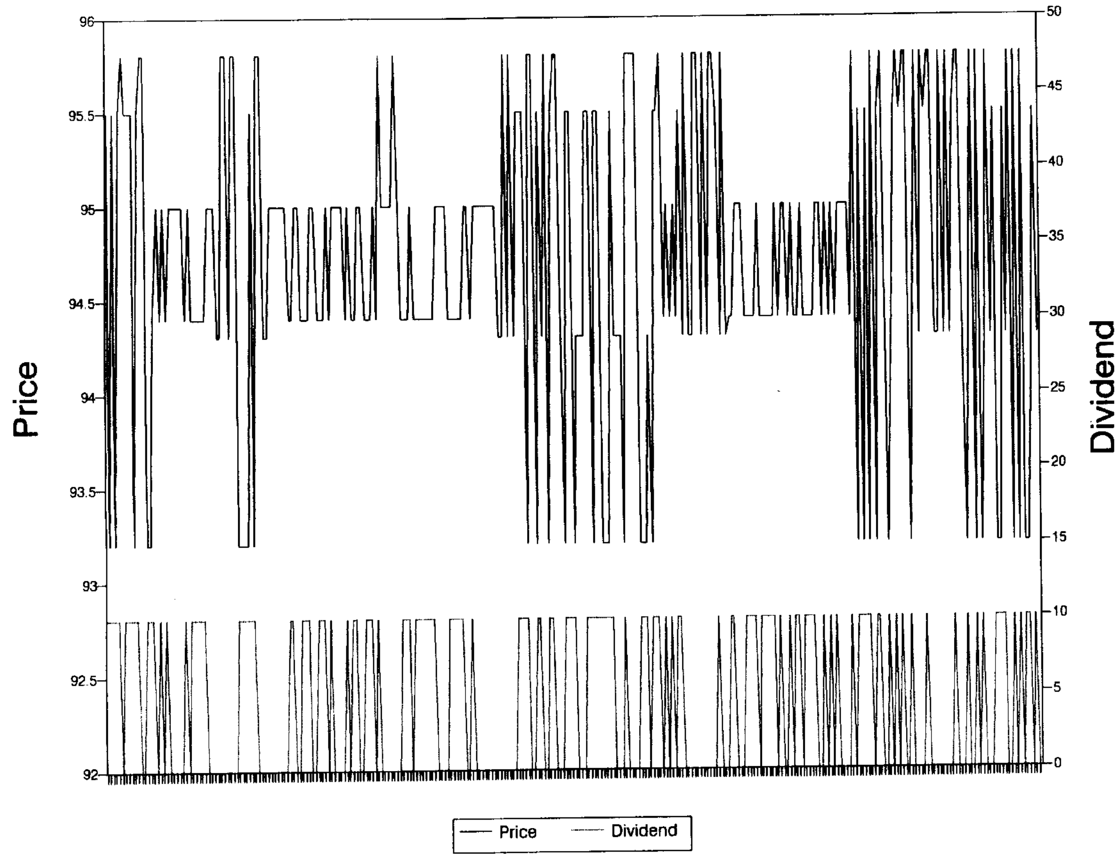

Figure 2

these are also random, I plot them at the bottom. It is clear that over time the model exhibits drastic changes in price volatility but there are only two volatility regimes: one is a *high volatility regime* and the second is a *low volatility regime*. Both regimes exhibit substantial persistence in the sense that once a regime starts it continues for some time until some unobserved factor causes the volatility regime to change. Variations in the growth rate of earnings does have a slight effect on these regimes so that within the high and low volatility regimes there are sub-regimes whose volatility depends to a small degree upon earnings.

C.4 Problem D: Volatility of the Foreign Exchange Rate and the Forward Discount Bias

Table 3 reports selected results of the international model which I now draw upon for the first time. Before discussing those let me define exactly the concept of "forward discount bias" which was mentioned in Problem D above. Suppose you estimate a regression of the form

$$\frac{ex_{t+1} - ex_t}{ex_t} = c + \beta(r_t^D - r_t^F) + \varepsilon_{t+1}$$

where $(ex_{t+1} - ex_t)$ is the change of the exchange rate between date t and date t + 1 while $(r_t^D - r_t^F)$ is the difference between the short term nominal interest rates in the domestic and the foreign economies. Under rational expectations the differential of the interest rates between the two countries at date t should provide correct predictor of the *actual* depreciation of the currency that will occur between date t and date t + 1. This means that apart from a technical correction for risk aversion the parameter β should be close to 1. In 75 empirical studies in which equations like the above were estimated, the estimates of the parameter β are significantly less than 1. Indeed, Froot [1990] estimates that the average for all these studies is -.88! The failure of this parameter to exhibit estimated values close to 1 has come to be known as the "forward discount bias" (see Engel [1996] for an extensive recent survey and Froot and Thaler [1990] for a simple exposition of the problem).

Table 3 reports (i) an interval in which the exchange rate fluctuates 95% of the time and where the mean exchange rate has been *arbitrarily* calibrated to be 120, (ii) the value of the parameter β which the simulation models predict. The selection of 120 as the mean of the exchange rate has no significance to the volatility measures reported. It is only meant to establish a comparable frame of reference. The actual rate of exchange

Table 3: The Volatility of the Exchange Rate and the Forward Discount Bias

	Under Rational Expectations	Under Rational Beliefs	Actual Yen/Dollar
Interval in which the exchange rate fluctuates 95% of the time	[115 , 125]	[67 , 173]	[84 , 156]
β - the forward discount bias parameter	.957	.152	Diverse

has fluctuated in part due to different inflation rates in the U.S. and Japan and I have thus computed the variance of the exchange rate based *on logarithmic detrending of the data*. The "actual" variability in the table is then that part of the variability of the Yen/Dollar exchange rate *around the average geometric trend*. The table exhibits the problems which arise under rational expectations: the variance of the foreign exchange rate is negligible and the parameter β takes a value close to 1. Under rational beliefs the results are drastically different: the variance of the foreign exchange is of the order of magnitude of observed fluctuations in the market. Finally, the forward discount bias parameter in the RBE reported in the table is .152 which is significantly less than 1. Within the class of models used here a negative parameter could not be predicted.

IV. Simple Explanations of How the Theory Resolves Each of the Four Problems

In Section III I demonstrated that the unified paradigm offered by the theory of rational belief equilibrium (RBE) goes a long way towards solving the four problems that could not be solved within the prevailing rational expectations paradigm. In this section I will offer a simple but systematic explanation of the results presented in Section III. In doing so I will also demonstrate the workings of the model of RBE.

(i) *Volatility of Prices and Exchange Rates*. The explanation of why the volatility of prices and exchange rates in an RBE exceed the level determined by the exogenous fundamentals of the economy is simple. Each agent forms his own theory of what the future will bring and the distribution of the private models in the economy constitute the "social state of belief." Variability in the state of belief in the market is then an important factor, together with the exogenous shocks, in explaining price volatility. Since the social state of belief is not observable we need to seek proxies for it. Incomplete proxies can be seen in the distribution of price forecasts announced by different forecasters for the market as a whole or for individual securities. Interesting distributions of short term and long term interest rate forecasts by professional economists is also revealing since all use the same data. Thus you may think of the "state of belief" in the market as a "distribution of beliefs".

Endogenous uncertainty is then the component of price volatility which is caused by the distribution of beliefs of the agents and therefore equilibrium price volatility can be represented as

Market Uncertainty = Exogenous Uncertainty + Endogenous Uncertainty

Since exogenous uncertainty is that component of market volatility which is determined by the volatility of the exogenous fundamental conditions in the market, it is then clear why total market volatility exceeds the level justified by fundamentals.

Without introducing technical details I stress that endogenous uncertainty has a dual effect on market volatility. One component of endogenous uncertainty is the *amplification* of the effect of fluctuations of exogenous fundamentals on price volatility. This is the effect whereby the distribution of beliefs in the market can cause fundamental "news" to have a larger effect on price volatility than would be true in a corresponding rational expectations equilibrium (where all traders have the same, correct, belief). The second component of endogenous uncertainty arises from the fact that *variations in the distribution of beliefs* cause additional price volatility which is unrelated to any fundamental "news." This component of endogenous uncertainty may have dramatic effects on the volatility of prices in an RBE since this component turns out to be affected by correlation and commonality of beliefs among traders. When a large number of agents become optimistic about capital gains, prices may rise. Conversely, when a large number of agents become pessimistic prices decline. Such variations need not be related to any fundamental news.

Endogenous Uncertainty also provides a natural explanation of the phenomenon which is recognized as "*market overshooting*." This is usually a reference to the fact that when prices are high they often proceed to go higher than can be justified by fundamentals and when they go low, they go lower than can be justified by the exogenous variables. Naturally, excess volatility and overshooting is part of the historical record and is incorporated in the empirical distribution of any market. Rational agents recognize this reality and incorporate overshooting in their own beliefs.

(ii) *The Forward Discount Bias in foreign exchange rates*. To see why this bias arises naturally in an RBE recall the rational expectations argument in favor of $\beta = 1$ (apart from the correction for risk aversion which I ignore here). Hence, in such an equilibrium it is a *theoretical* conclusion that the difference between the one period nominal rates in the two countries at date t is exactly equal to the expected percentage depreciation of the exchange rate between the two currencies between dates t and $t + 1$. This *expectational* argument implies that in the real economy the differential between the one period nominal rates in the two countries will be an unbiased *statistical* forecast of the one period depreciation of the exchange rate in the next period. Under this proposition one would expect to have a regression coefficient of 1 between the percentage differential of the nominal rates at date t and the *actual* percentage change of the exchange rate between dates t and $t + 1$.

The theory of RBE predicts that agents holding rational beliefs will make significant forecasting mistakes. This would result in a true, equilibrium, process of the exchange rate which would fluctuate excessively in part due to these mistaken forecasts. Hence, at almost no date would the nominal interest differential between the two countries be an unbiased estimate of the rate of depreciation of the exchange rate one period later and under such circumstances one should not expect the regression coefficient to be close to one. Agents who want to take advantage of such a regression, basing their investment strategy on a nominal rate differential which appears to offer an arbitrage opportunity, will find that this is not arbitrage in the standard riskless sense of the term. Since in an RBE the change of the exchange rate at date $t + 1$ *is a random* *variable* at date t any attempt to arbitrage between date t and date t + 1 requires agents to take the risk that the statistical regression model does not apply to the circumstances which prevail in the market *at the time* in which they plan to invest.

Should we expect that under rational beliefs the parameter β satisfies $\beta < 1$? The answer is yes for the following reason. Consider first a rational expectations equilibrium in which the difference between the domestic and foreign nominal interest rates is z%. In that equilibrium you do not need to form expectations on the currency depreciation itself. It is sufficient for you to believe that other investors or currency arbitrageurs know the true probability of currency depreciation and they have already induced the interest differential to be equal to the average rate of currency depreciation *which will be z%.* Now consider an RBE. All agents know that no one knows the true probability distribution of the exchange rate and therefore the exchange rate is subject to endogenous uncertainty. Being risk averse, agents who invest in foreign currency would demand a risk premium on endogenous uncertainty and over the long run the difference $(1- \beta)$ is the premium received by currency speculators for being willing to carry foreign currency positions. For a positive premium it follows that $\beta < 1$.

(iii) *The GARCH Property of Asset Prices*[8]. The explanation of the GARCH property requires the understanding of one more property of economic dynamics, namely, "persistence over time." Persistence is the property according to which the probability at date t of an event occurring at date t + 1 is higher when the event occurred at date t compared with the probability given that the event did not occur at date t. It is well known that many economic variables exhibit persistence and I shall now explain why the GARCH property of prices is a consequence of the persistence in the state of beliefs of the investors.

As indicated earlier, the states of belief of different individual investors may be highly correlated and this is a consequence of the many modes of communication in our society. Investors talk to each other and this interaction causes them to influence each other; they all read the same newspapers, the same reports of the Wall Street analysts and watch the same television programs which feature expert views on the economic conditions in the future. The analysts and experts know each other, they talk to each other and attend the same conferences thus tend to correlate their views either in agreement or disagreement. The consequence of this correlation among the beliefs is that the distribution of beliefs tends to switch across different "cognitive" centers of gravity.

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[8] For more details about the nature of GARCH and related processes see Bollerslev, Chou and Kroner [1992] and Bollerslev, Engle and Nelson [1994].

Indeed, each such center of gravity is a "belief regime". The important examples of such regimes of belief are regimes of "consensus" and "non-consensus." It turns out that what really matters for the emergence of the GARCH phenomenon is the *persistence* of the regime of consensus vs. the regime of non market consensus. A regime of *market consensus* is formed when the models of the majority of traders generate similar predictions and if the regime persists, then over time the traders move *together* between states of optimism and states of pessimism. Such fluctuations between optimistic and pessimistic outlook on the news and on market performance may occur on many different frequencies. *Non-consensus* is a belief regime in which the distribution of models used by the agents is relatively spread out and consequently their predictions vary widely across the different possible outcomes in the future. If the regime of non-consensus persists then the diverse forecasts tend to cancel each other out over time.

Putting together all the parts laid out above I now observe that since the distribution of beliefs tends to persist, when a regime of consensus is formed the volatility of security prices will be high. This is true because when the consensus has an optimistic outlook they all seek to buy the same securities and when the majority adopts a pessimistic outlook they all seek to sell the same securities. Conversely, when a non-consensus regime occurs, the opposite is true: now the distribution of beliefs remains relatively fixed leading to a regime of low volatility since the excess demands of the optimists cancel the excess demands of the pessimists.

To generalize these conclusions beyond the simulation models, the theory of RBE shows that the variance of stock prices depends upon the *distribution of beliefs* in the market and since this distribution changes over time, so does the variance of stock prices. Also, in an RBE investors can utilize observed information ("news") and realized prices to determine their state of belief about the future. Consequently, the distribution of beliefs and hence the variance of prices may depend upon both the correlation among the models of the agents as well as the observed fundamental news and realized prices. Since either or both of these may change abruptly, so can the induced regime of beliefs.

The models used in the simulations reported here relate to events which occur over relatively long stretches of time and hence the simulation results apply to low frequencies (i.e. months and years). These do not address the structure of volatility at high frequencies investigated by some papers of the GARCH literature (see for example Bollerslev, Chou and Kroner [1992] and Brock and LeBaron [1996]). This limitation of the results here should not obscure the main conclusion to which the theory of RBE leads: the GARCH phenomenon is caused both by the persistence as well as by the abrupt shifts in the distribution of beliefs. In turn, the dynamics of the distribution of beliefs has two features. First, the shifts of the distributions are the consequences of the correlation among the states of beliefs of the individual agents, and second, the persistence of each volatility regime is a result of the persistence in the distributional regime as described above. These forces hold over low or high frequencies.

(iv) *The Equity Premium and the Riskless Rate*[9]. Explaining the factors which determine the equity risk premium (i.e. "the" premium) in an RBE is ultimately simple but demands the review of the technical conditions which formulate the rationality of beliefs of the agents. A *direct* and simple explanation flows naturally from the resolution of Problem A. It proposes that in an RBE endogenous uncertainty causes the total level of uncertainty to exceed the level that would prevail under rational expectations. Risk averse investors would then demand a higher risk premium for holding equity which is more risky in an RBE than in a rational expectations equilibrium and for that reason the premium would be higher in an RBE. This explanation has a grain of truth but needs to be qualified by two additional considerations.

The first consideration suggests that due to the diversity of beliefs the equity premium arises in a world where optimists and pessimists reside together. The risk premium demanded by optimists is likely to be different from the premium demanded by pessimists and hence, the market premium must depend upon the *distribution* of beliefs. Indeed, there are proportions of optimists and pessimists which do not generate a higher equity risk premium than is generated under rational expectations. Second, an important component of the equity premium puzzle has been the question of why the riskless rate predicted by rational expectations models has been so much higher than the mean riskless rate realized over the last century and this question must be cleared as well. The *direct* explanation given above does not address the question of why the riskless rate is so much lower in the simulated RBE relative to rational expectations equilibria.

To gain intuition into the two issues above I must bring you into some of the more technical aspects of the theory and to do that I examine a very simple model (based on Kurz [1997d]). Consider an economy with two types (α and β) of agents who are different only in their models of market price behavior (i.e. their beliefs). As part of their models, each of the type α agents has a random variable called "an assessment" and when the assessment takes the value 1 the agent uses probability distribution F_1 of future prices and when it takes the value 2 the agent uses probability distribution F_2. These assessment variables are different for the two types. For this reason I denote the probabilities used by type β agents by G_1 when the assessment of a type β agent takes the value 1 and by G_2 when the assessment of a type β agent takes the value 2. However, I also assume that

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[9] The debate regarding "The Equity Premium Puzzle" was initiated by Mehra and Prescott [1985]. A sample of other papers on the subject include Mankiw [1986], Reitz [1988], Weil [1989], Epstein and Zin [1990], Constantinides [1990] and Campbell and Cochrane [1995].

there is a very large number of agents of each type and each of them has his own separate assessment. Now, the assessments of the large number of agents of each of the types are, statistically speaking, the *same* random variables since these agents are of the *same* type but now comes the deeper question: are these assessments *independent random variables*? To address this question I must take an indirect route.

One criticism of the theory of rational beliefs has suggested that in a large economy consisting of many agents with *independent* beliefs the law of large numbers would operate to average out the diversity of beliefs. Such averaging should render the model of diverse beliefs irrelevant, leading the model of a large economy to function like a model of the representative household with a *single*, rational expectations belief. This "intuitive" argument is false and the reasons why it is false are the key to understanding why a large equity premium and a low riskless rate can be generated in an RBE.

Let me then return to my simple model and make the strong assumption that all the assessments within each type are i.i.d. with the probability of assessment taking the value of 1 being, say, .60. The consequence of this assumption is that although the probability used by any one agent depends upon his assessment, the distribution of beliefs in the economy is fixed at ((.60, .40), (.60, .40)). That is, *at all times* 60% of type α agents use probability distribution F_1 and 40% of them use F_2. A similar situation is assumed with respect to type β agents. If I now interpret F_1 and G_1 to mean "optimistic beliefs about higher returns next period" and F_2 and G_2 to mean "pessimistic beliefs about higher returns next period" then I have an economy where the law of large number holds as required. At *all times* the distribution of beliefs is constant with 60% of each type optimistic and 40% pessimistic.

I need to specify what I mean by the term "optimistic". To do that let the long run frequencies which agents can compute from the history of the economy be denoted by Γ. Then I use the term "optimism" to mean that the *proportions* between the probabilities of higher prices at date t + 1 specified in F_1 and those probabilities specified in Γ are larger than 1. Similarly for the optimistic G_1 in relation to Γ. I use the term "pessimism" to mean that the *proportions* between the probabilities of higher prices at date t + 1 specified in F_2 and those probabilities specified in Γ are smaller than 1. Similarly for the pessimistic G_2 in relation to Γ. I will call these proportions (which are fixed for each type but may be different across the two types) the "intensities of optimism" or the "intensities of pessimism". I use the term "intensities" rather than "intensity" since these intensities of optimism or pessimism may vary depending upon current market prices.

I selected the numbers in such a way that in this economy 60% of the agents are always optimistic, using F_1 or G_1, and hence each individual agent fluctuates between optimistic and pessimistic outlooks with a frequency of .60 in the optimistic mode and a frequency of .40 in the pessimistic mode. This would make sense only when I consider the rationality of belief conditions which the agents must satisfy. These stipulate that the beliefs may fluctuate over time but on average must correspond to Γ. The RBE is then established if all type α agents satisfy the rationality of belief condition $.60F_1 + .40F_2 = \Gamma$ and $.60G_1 + .40G_2 = \Gamma$ for type β agents . But now I need to compare two equilibria: an REE in which *all the agents hold the belief* ′ and the RBE in which *60% are optimists and 40% are pessimists relative to* ′. I claim that these two are very different equilibria with drastically different equity premia and volatility characteristics.

To convince you of that fact suppose that the initial percentage of pessimists in the economy is $x = .40$ and in equilibrium the rationality condition $(1 - x)F_1 + xF_2 = \Gamma$ is satisfied. Now I lower the percentage $x = .40$ to x'. Would the rationality of belief condition $(1 - x')F_1 + x' F_2 = \Gamma$ be satisfied with x'? The answer is no since my decrease of the percentage of pessimists from $x = .40$ to x' without changing the matrices (F_1, F_2, G_1, G_2) means that I reduced the weight assigned to the pessimistic matrix F_2 and increased the weight assigned to the optimistic matrix F_1 leading to the result that $(1 - x')F_1 + x' F_2 \neq \Gamma$. Hence, as the number of pessimists in the market declines, I must adjust the intensity parameters in F_2 and in G_2 so that the intensity of their pessimism *increases*. Indeed, a point will be reached at which I could not lower the fraction of pessimists any further since the intensity of their pessimism has reached a point where, given some price, they are virtually certain that they will lose money between date t and date t + 1. I will then have an economy with a reduced proportion of pessimists but who are so intensely pessimistic that they are willing to pay a very high price for the bill to secure their wealth for next period. What will happen to the interest rate and to the risky returns in the model under these circumstances? The price of the bill will rise, lowering the riskless rate, and the price of the stock will fall causing the equity risk premium to rise. Finally, as realized prices vary over time, the degree of optimism and pessimism may change with prices, leading to fluctuations in the number and intensity of the pessimists and optimists in the market. Such fluctuations induce a level of volatility which may be dramatically higher than the volatility of the corresponding REE. This concludes my demonstration that the RBE under discussion is very different from the REE with a representative agent.

The central observation is that the rationality of belief conditions are linear conditions of the form $(1 - x)F_1 + xF_2 = \Gamma$ but variations in the percentage/intensity combinations of optimists and pessimists have a non-linear impact on the demand functions for securities. Hence, as these combinations vary over the feasible parameter space of the model, the riskless rate and the equity premium change. For configurations of a small proportion of optimists with a high intensity level of optimism the demand for borrowing will be high and hence the riskless rate will rise. Such combinations may lead to a high premium with a high riskless rate. The unique combination of *a high premium* *with a low riskless rate arises only when the pessimists are in the minority but the intensity of their pessimism is high.* This *specific* conclusion depends upon the structure of the real economy as stipulated by the profit process paid by the stock.

Given the basic observation that at any date the risk premium is determined by the exogenous variables and by *the distribution of beliefs* in the market, I reexamine the assumptions made earlier. Recall that I have assumed that the assessments are i.i.d. in order to refute the criticism that heterogeneity of beliefs is irrelevant in a large economy with independent beliefs. Extensive research conducted in recent years has shown that it takes very little local interaction among agents in the market in order to remove the effect of the law of large numbers on equilibrium variables such as prices. More specifically, under small local interactions, equilibrium aggregate variables of a large economy *act as random variables* rather than as constants[10]. Given the natural interaction among the agents in financial markets there is ample theoretical justification for assuming that the beliefs of agents in the market are correlated and hence their assessments are not jointly i.i.d. On the empirical side there is substantial evidence that the distribution of beliefs in the market shifts over time jointly with prices implying that individual beliefs are correlated. Hence, both theoretical as well as empirical arguments imply that we should study models where the distribution of beliefs is a random variable, jointly distributed with prices and other equilibrium variables.

The argument developed earlier (for an economy with i.i.d. assessments) regarding the belief intensity of the pessimists remains valid in an economy with correlation among the assessments of the agents. The only difference is that now the distribution of beliefs changes over time and the riskless rate and equity premium vary with the states of the economy. Hence, the RBE model's prediction of the long term averages of the riskless rate and of the equity premium depends now also upon the frequency at which the system visits those distributions of beliefs which generate low riskless rate and high premium. Keep in mind that the available empirical evidence consists of the moments of the empirical distribution of the rates of return on assets and the premium. Hence, the model's ability to explain the available empirical evidence regarding the riskless rate and the equity premium is enhanced by the potential correlation among the states of the economy.

In sum, the RBE theory presented here offers a very simple explanation for the observed low average riskless rate of around 1% and a high equity premium of about 6%. The theory proposes that such a pattern arises as a consequence of the diversity of beliefs

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[10] See, for example, the papers by Brock [1993],[1996], Durlauf [1993],[1994] and Föllmer [1974].

in our financial markets when the majority of traders are optimistic but where there is always a minority of *intense* pessimists. The identity of these agents changes at all times since no rational agent is always optimistic or always pessimistic. This distribution of beliefs has two drastic consequences. First, it causes our financial markets to "overshoot" in the sense of experiencing much larger fluctuations of prices than could be explained by exogenous, fundamental, factors. Second, and this is the main conclusion of this Section, the high intensity of the pessimists is the decisive factor which, in the long run, dominates the market for short term debt instruments. These are the agents who push the riskless rate down and the equity premium up. This ability of the theory of Rational Beliefs to provide this explanation of the empirical evidence is a central dimension of the unified paradigm proposed in this paper. That is, our explanation of the empirical evidence flows directly from the conditions of rationality of the agents since the crucial asymmetry of the argument which grants the pessimists the greater intensity is a direct consequence of the rationality conditions.

A final observation regarding the historical record, is of interest. There is some evidence that the riskless rate has exhibited a rather irregular pattern over the last 200 years. Table 4, drawn from data provided in Siegel [1994], shows that the very low average rate of return of less than 1% on riskless debt instruments is a phenomenon

Table 4: Real Rates of Return on Debt Instruments

	on Short Term Government	on Long Term Government
1802-1870	5.1%	4.8%
1871-1925	3.2%	3.7%
1926-1997	0.6%	2.0%

which occurred mostly after the great depression. Indeed, Siegel [1994] shows that the large spread between rates of return on long and short term government debt instruments opened up exactly around 1930 and remained high until 1997. I might caution the reader that historical data prior to World War I are subject to large errors and could be interpreted in many different ways. Suppose, however, that Siegel [1994] is correct in identifying the data on the riskless rate. In that case, it appears that the 1930 depression has something to do with the low riskless rate. But such a fact provides further support for the theory offered in this paper since this may establish the fact that the pessimists in my RBE model based their pessimism on the experience of the 1930's. This does not mean that the probability which the pessimists attached to capital losses are exactly the empirical frequencies of the great depressions. These empirical frequencies are part of the average historical record in the matrix Γ. Rationality of belief permits the pessimists to hold a probability F_2 or G_2 which do not correspond to any specific empirical frequency. However, it is the great depression that may have been responsible for the *nature* of the RBE which we have been discussing all along.

Appendix

Two Econometric Studies About the U.S. Stock Market Supporting the Theory

All studies reported in the text dealt with the four Problems described in Section I and utilize the methodology of *simulating models* of RBE to study these problems. An alternative way of testing the theory and evaluating its empirical implications is to conduct *econometric studies* utilizing data from actual financial markets. This type of research aims to quantify the magnitude of endogenous uncertainty as a component of aggregate market uncertainty and to test some of the predictions of the theory. Since the central topic of this paper is the effect of endogenous uncertainty on market volatility, these studies can provide the reader with some added understanding of the theory. For this reason I now briefly review the results of two such studies of the U.S. stock market which are included in Kurz (ed.)[1997][11].

In Kurz [1997b] I study the behavior of excess returns on the Standard and Poor's 500 Composite Index in the context of a model of structural change as represented by a sequence of "regimes" in the U.S. economy between 1947 and 1992. Each regime emerges spontaneously at a random date and within each regime the structure remains fixed except, possibly, for a simple time drift. Each regime is then terminated randomly.

Models of regime switching are standard in the literature. I therefore decompose the post-war period of 1947:1-1992:3 into three subperiods: 1947:1-1965:4, 1966:1-

[11] See Kurz, M. [1997b] "Asset Prices with Rational Beliefs" which is Chapter 9 and Kurz, M. and Beltratti, A. [1997] " The Equity Premium is No Puzzle" which is Chapter 11 of Kurz (ed.)[1997].

1981:4 and 1982:1-1992:3. The time span of each of these regimes is long enough to enable statistical analysis with some measure of reliability: within a regime the structure remains relatively constant and consequently it is possible to estimate its parameters with some degree of confidence.

The object of the study is then to construct, in retrospect, a statistical model which would explain the movement of excess returns in terms of the information available at the time of decision. More specifically, I examine how excess returns behaved differently in the three specified subperiods and which of the variables that were known to agents at the starting date of each period of measurement (the study explores one and two year returns) could statistically explain excess returns within each subperiod. Since the study is done *in retrospect* we could look back at history and discover that within each subperiod there was information, *including the knowledge of the regime itself*, that could have explained and predicted returns. The fact is that most agents failed to do so since at the time they did not have the knowledge or sufficient statistical evidence to arrive at those conclusions which we can see today. Note, however, that this is exactly the way in which the mistake functions of the agents are statistically discovered in retrospect. Since the mistake functions provide the mechanism for generation of endogenous uncertainty, I will show now how estimation of the mistake functions can provide an estimate for the effect of endogenous uncertainty on market volatility.

According to the market efficiency theory the equity returns of a market index vary due to two factors: (i) variations in the forecasted component of returns which are due to exogenous variables and (ii) pure noise. Indeed, after correcting for the forecasted return, the process of excess returns is a random walk and it is appropriate to regard it as the *pure* volatility of asset prices. The theory of RBE asserts that there is a third component to the variability of returns: the effect of the mistakes of the agents in generating endogenous uncertainty. Although this component can be estimated in retrospect, such estimation is not precise and hence part of the unexplained pure noise may be internally propagated endogenous uncertainty which cannot be distinguished from the exogenously generated true pure noise.

The model formulation demonstrates that the growth rate of consumption adjusted for risk aversion is a measure of the forecasted returns as in (i) above and it is constructed as an instrumental variable. I call this variable the "forecasted returns" and refer to the difference between actual returns and the "forecasted returns" as the "excess returns." The central idea of the estimation procedure is to show that due to the mistakes of the agents, a portion of these excess returns is explainable in retrospect in terms of variables that were observable at the time. The fraction of excess returns which is explained by these variables is then the lower bound on the contribution of endogenous uncertainty to total volatility of excess returns. The variables used to demonstrate this fact are of two types. The first type are variables which specify the start and end dates of the three postulated regimes and which are called "the regime variables." Second, I use six variables in the estimation of the model: pure time drift; one and two period lagged rates of change of basic commodity prices; one and two period lagged rates of growth of manufacturing output in the U.S. and the lagged equity return itself. For brevity I call these six variables "the explanatory variables."

Table A-1 reports the accuracy of explaining returns in terms of the adjusted R^2 of the estimated model. The first row records the contribution of "forecasted returns" (i.e. the consumption growth instrument). The second row estimates the contribution of the six "explanatory variables" and the third estimates the net effect of all the regime variables and their interactions with the six variables. Interpreting the pure variability of excess returns as represented by that part which cannot be explained by "forecasted returns" as in (i) above, then this fraction is .79 in the case of one year returns and .75 in the case of two year returns. Table A-2 decomposes these amounts into two parts, one fraction representing the lower bound of the internally propagated endogenous uncertainty. I use the term "lower bound" since this component includes only the volatility that could be accounted for by the variables in the model. Thus, for example, the 64% in the table is calculated by taking $64\% = \frac{.73 - .25}{1.00 - .25}$. The second part is pure noise but as explained earlier, part of this noise may also have been internally propagated but it is not clear how to separate it from the exogenously generated noise. I then conclude that

Table A-1
The Accuracy of Explaining One and Two Year Returns
(measured in terms of adjusted R^2)

	one year returns	two year returns
When using only "forecasted returns" measured by growth of consumption	.21	.25
When adding "the explanatory variables" but no interaction with regime variables	.27	.32
When adding "regime variables"and allowing full interaction	.61	.73

at least 50% of the risk of one year excess returns and 64% of the risk of two year returns are internally propagated and are classified as endogenous uncertainty.

Table A-2
Decomposition of the Risk of Excess Returns

	one year returns	two year returns
Endogenous uncertainty: internally propagated risk (fraction)	51%	64%
Pure noise (fraction)	49%	36%

The study reported in Kurz and Beltratti [1997] uses the same techniques used above except that it studies the asset allocation of mutual funds. The study estimates the mistake functions of the managers of 63 major U.S. mutual funds which were classified as "Balanced" or "Income and Growth." It covered the period 1982:4 - 1995:1 and examined only the mix of "Equities", "Bonds" and "Cash". The paper estimates mistake functions with statistically significant parameters for a large number of funds. Indeed, the paper shows that the method of estimating mistake functions for funds can provide a powerful tool for the evaluation of the performance of mutual funds by identifying poor performance which resulted from "luck" as distinguished from such performance which resulted from poor "judgement."

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